Lifting Scheme Generalization for Lossless Image/Signal Compression by Data Dependent Locally Adaptive Wavelet Transform

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ABSTRACT

A new algorithm of locally adaptive wavelet transform is presented. The algorithm implements the integer-to-integer lifting scheme. It performs an adaptation of the wavelet function at the prediction stage to the local image data activity. The algorithm is based on the proposed generalized framework for the lifting scheme that permits to obtain easily different wavelets in the case of the ($\widetilde{N}$, N) lifting.

New wavelets were derived analytically and were used in the compression algorithm, which performs the adaptation to the local data activity by the hard switching between (2, 4) and (10, 4) lifting filter outputs according to an estimate of the local data activity. When the data activity is high, i.e., in the vicinity of image edges, the (10, 4) lifting is performed. Otherwise, in the plain areas, the (2,4) wavelet decomposition coefficients are calculated. The calculations are rather simples that permit the implementation of the designed algorithm in fixed point DSP processors. The proposed adaptive transform possesses the perfect restoration of the processed data and possesses good energy compactation. The designed algorithm was tested on different images. The proposed adaptive transform algorithm can be used for the loss-less image/signal compression.

Key words:
image processing, lifting scheme, lossless compression, wavelets

I. INTRODUCTION

In the past decade, the wavelet transform has become a popular, powerful tool for different image and signal processing applications such as noise cancellation, data compression, feature detection, etc. Meanwhile, the aspect of fast wavelet decomposition/reconstruction implementation, especially for image compression applications, now continues to be under consideration.

The first algorithm of the fast discrete wavelet transform (DWT) was proposed by S.G.Mallat [1]. This algorithm is based on the fundamental work of Vetterli [2] on signal/image decomposition by 1-D quadrature-mirror filters (QMF), and orthogonal wavelet functions. The next breakthrough concerning a fast DWT algorithm implementation was made by W.Sweldens [3] who first proposed the lifting scheme. His paper written with I. Daubechies [4] was dedicated to integer number processing in the lifting scheme giving the possibility of a further improvement of DWT for signal/image compressing algorithms and significant simplification of the hardware implementation.

In this paper, we present an algorithm for 2D DWT with the high-pass filter that is adaptive to the image local data activity. The algorithm is based on the integer lifting scheme that permits to construct easily the wavelet-based FIR filters with the different properties varying the order of the filter and the filter coefficients. This way, the image decompositions can be optimized to achieve a minimum of the entropy in the wavelet domain.

The proposed algorithm uses the generalized approach [5] to construct the different wavelets, which we extended to the case of the arbitrary orders of the predictor and update filters. This way, new wavelets were derived analytically and were used in the image compression algorithm to obtain the better results with the adaptive lifting DWT.

The knowing problems dealing with the synchronization at the decomposition and restoration stages [6, 7] are resolved using the simple local data activity estimator, which uses the non-lifted data to decide is the current pixel belongs to the plain area or to the vicinity of an object edge.

II. GENERALIZATION OF THE LIFTING SCHEME

The lifting scheme is widely used in the wavelet based image analysis. Its main advantages are: the reduced number of calculations; less memory requirements; the possibility of the operation with integer numbers. The lifting scheme consists of the following basic operations: splitting, prediction and update.

Splitting is sometimes referred to as the lazy wavelet. This operation splits the original signal $\{x\}$ into odd and even samples:

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$$s_l = x_{2l},\ d_l = x_{2l+1}. \quad (1)$$

Prediction, or the dual lifting. This operation at the level k calculates the wavelet coefficients, or the details $\{d^{(k)}\}$ as the error in predicting $\{d^{(k-1)}\}$ from $\{s^{(k-1)}\}$:

$$d_l^{(k)} = d_l^{(k-1)} + \sum_{j=-\tilde{N}/2}^{\tilde{N}/2} p_j s_{l+j}^{(k-1)}, \quad (2)$$

where $\{p\}$ are coefficients of the wavelet-based high-pass FIR filter and $\tilde{N}$ is the prediction filter order.

Update, or the primal lifting. This operation combines $\{s^{(k-1)}\}$ and $\{d^{(k)}\}$, and consists of low-pass FIR filtering to obtain a coarse approximation of the original signal $\{x\}$:

$$s_l^{(k)} = s_l^{(k-1)} + \sum_{j=-N/2}^{N/2} u_j d_{l+j}^{(k-1)}, \quad (3)$$

where $\{u\}$ are coefficients of the wavelet-based low-pass FIR filter and N is the prediction filter order.

Sometimes, the normalization factors can be applied to the wavelet details and approximations to produce the transformed data values more similar to the "classic" DWT algorithm.

The inverse transform is straightforward: first, the signs of FIR filter coefficients $\{u\}$ and $\{p\}$ are switched; the inverse update followed by inverse prediction is calculated. Finally, the odd and even data samples are merged.

A fresh look at the lifting scheme first was done in [5], where the FIR filters that participate in the prediction and update operation are described in the domain of Z-transform. According to this approach, the transfer function of the prediction FIR filter can be formulated as follows

$$H_p(z) = 1 + p_0\left(z + z^{-1}\right) + p_1\left(z^3 + z^{-3}\right) + \ldots$$
$$\ldots + p_{\frac{\tilde{N}}{2}-1}\left(z^{\tilde{N}-1} + z^{-\tilde{N}+1}\right) \quad (4)$$

The $H_p(z)$ must has zero at $\omega = 0$, i.e., at $z = 1$. can be easily found [5] that this condition is satisfied when

$$\sum_{l=0}^{\frac{\tilde{N}}{2}-1} p_l = -\frac{1}{2}. \quad (5)$$

When the condition (5) is satisfied, $H_p(-1) = 2$ and $H_p(0) = 1$ that means the prediction filter has gain 2 at $\omega = \pi$ and unit gain at $\omega = \frac{\pi}{2}$. Thus, if one want to obtain the normalized detail coefficients, the normalization factor must be equal to $\frac{1}{2}$.

Following this approach, the transfer function for update filter can be obtained. We prefer to formulate this transfer function in the terms of $H_p(z)$:

$$H_u(z) = 1 + H_p(z)\Big\{u_0\left[(z) + \left(z^{-1}\right)\right] + u_1\left[\left(z^3\right) + \left(z^{-3}\right)\right] + \ldots$$
$$\ldots + u_{\frac{N}{2}-1}\left[\left(z^{N-1}\right) + \left(z^{-N+1}\right)\right]\Big\} \quad (6)$$

Similarly, $H_u(z)$ must has zero at $\omega = \pi$, i.e., at $z = -1$. It can be easily found [5] that this condition is satisfied when

$$\sum_{l=0}^{\frac{N}{2}-1} u_l = \frac{1}{4}. \quad (7)$$

When the condition (7) is satisfied, $H_u(1) = 1$ and that means the prediction filter has gain 1 at $\omega = 0$.

An elegant conversion of the formulas (5), (7) in the case of (4,4) lifting scheme was proposed in [5] to reduce the degree of freedom in the predictor and update coefficients. With some modifications, the formulas for the wavelet filters coefficients are as follows:

$$p_0 = -\frac{128 + a}{256}, \quad p_1 = \frac{a}{256}, \quad (8)$$

$$u_0 = \frac{64 + b}{256}, \quad u_1 = -\frac{b}{256}, \quad (9)$$

where a and b are the parameters that control the DWT properties. Also, in [5] it was found the correspondence between these control parameters and the conventional (non-lifted) wavelet filters. Some results of this relationship we reproduce here in the Table 1.

Table 1. Relationship between parameters a and b in lifting scheme and conventional wavelet filters

a	b	Filter name	Lifting notation
0	0	5/3 bi-orthogonal	(2,2)
0	12	9/3 MPEG-4	(2,4)
16	8	13/7 Sweldens	(4,4)
16	16	13/7 CRF	(4,4)

The behavior of the lifting scheme with the respect of a and b values can be evaluated easily in the frequency domain using transfer functions (4), (6) with $z = e^{-j\omega}$. Figures 1 and 2 represent the magnitude of the frequency responses of high-pass and low-pass (2,2) and (4,4) filers with $a = 0$, $b = 0$ and $a = 16$, $b = 16$, respectively. These figures were obtained with MATHCAD200i software.

By simulations with MATHCAD2000i software, we found that better frequency response of (4,4) lifting filter can be obtained with the negative a and b parameters. In particular, when $a = 20$, the transition band of the high-pass filter is the narrowest. However, increasing more the coefficient a value leads to the appearance of the false side lobe in the low-frequency band. This fact is demonstrated in the Fig.3.

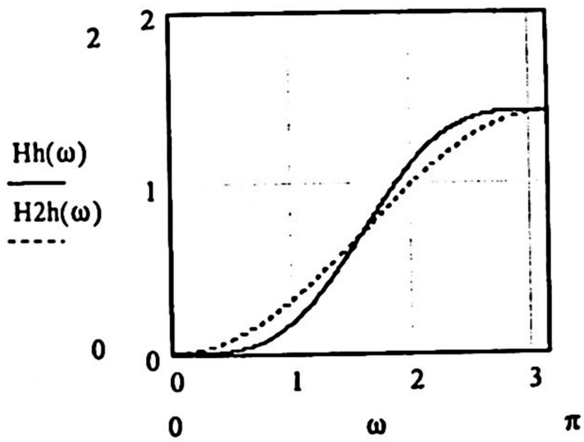

Figure 1. Normalized frequency response of lifting predictor with $\widetilde{N} = 2$ and $a = 0$ ($H2h(\omega)$, dotted line) and $\widetilde{N} = 4$ and $a = 16$ ($Hh(\omega)$, solid line)

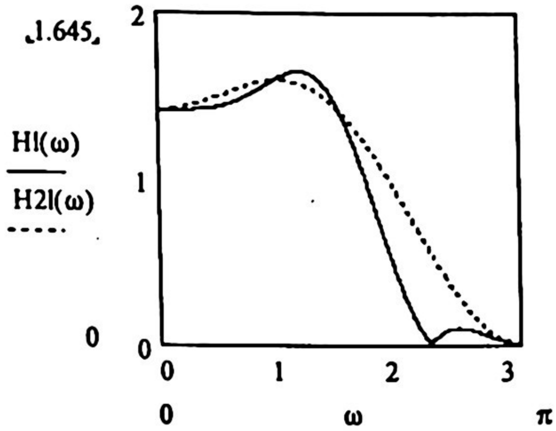

Figure 2. Normalized frequency response of lifting update filter with $\widetilde{N} = 2, N = 2$ and $a = 0, b = 0$ ($H2l(\omega)$, dotted line) and $\widetilde{N} = 4, N = 4$ and $a = 16, b = 16$ ($Hl(\omega)$, solid line)

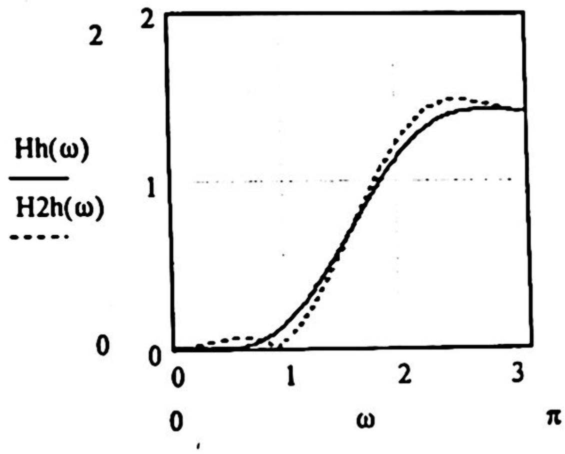

Figure 3. Normalized frequency response of lifting predictor with $\widetilde{N} = 4$ and $a = 20$ ($\mathrm{Hh}(\omega)$, solid line) and $a = 32$ ($H2h(\omega)$ dotted line)

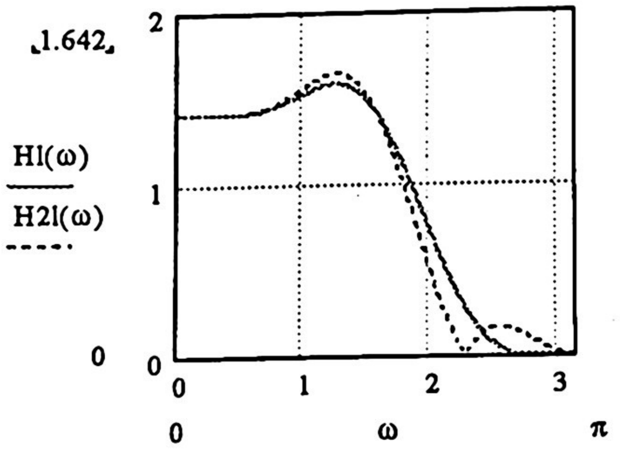

Figure 4. Normalized frequency response of lifting update with $\tilde{N}=4, N=4$, $a=20$ and $b=9$ ($Hl(\omega)$, solid line) and $b=18$ ($H2l(\omega)$, dotted line)

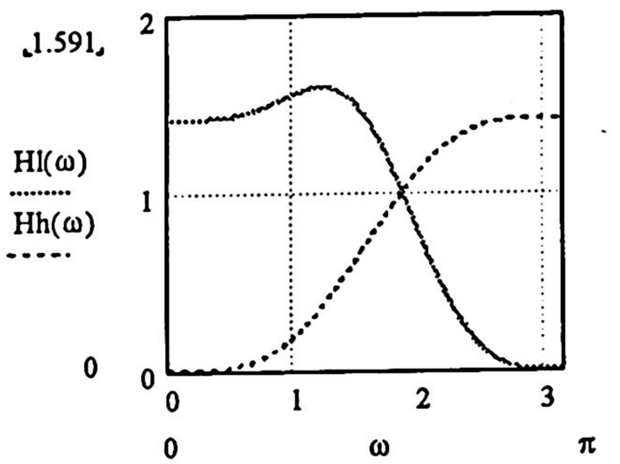

Figure 5. Normalized frequency responses of (4,4) lifting filters with $a=16$ and $b=8$

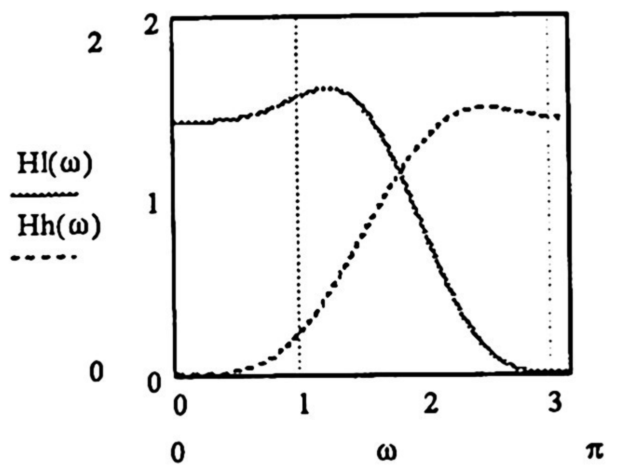

Figure 6. Normalized frequency responses of 13/7 wavelet filters

The behavior of update filter is more complicated. The frequency response of this low-pass filter depends not only on b parameter value, but inherently on the a value. This fact is explained by the nature of the lifting scheme. In practice, the influence of the coefficient a value (which is relatively small) can be avoided choosing this value to produce the predictor with the desired frequency response and then, choosing the appropriate value of the coefficient b. For example, if we choose $a=20$, the narrowest transition band in the update filter is obtained with $b=9$ meanwhile with the greater b a side lobe appears in the high frequency band. This fact is illustrated in the Fig.4.

The simulations with the test image "Lena" and "Mandrill" show that the better energy compactation properties with the (4,4) lifting filters are obtained with values $a=16$ and $b=8$ are optimal in the sense of the minimum entropy of the decomposed images. The obtained entropy values were 4.24 bpp (bits per pixel) for the image "Lena" and 6.28 bpp for the image "Mandrill". The resulted frequency responses of the (4,4) lifting filters are presented in Fig.5, and Fig. 6 shows the frequency responses of the correspondent "classic" wavelet filter pair (see Table 1).

Using the generalization of the lifting scheme (4)- (7), we found by simulations that the coefficients of the lifting filters of an arbitrary order higher than 4 can be found according to the recursive formulas:

$$p_0=-\frac{128+a}{256}\,,\quad p_1=-\frac{1}{2}-p_0\,,$$
$$p_2=-\frac{p_1}{c},\, p_1=p_1-p_2\,,\,\ldots,$$
$$p_{\tilde{N}}=-\frac{p_{\tilde{N}-1}}{c},\, p_{\tilde{N}-1}=p_{\tilde{N}-1}-p_{\tilde{N}} \qquad (10)$$

$$u_0=\frac{64+b}{256}\,,\quad u_1=\frac{1}{4}-u_0\,,$$
$$u_2=\frac{u_1}{d},\, u_1=u_1-u_2\,,\,\ldots,$$
$$u_N=\frac{u_{N-1}}{d},\, u_{N-1}=u_{N-1}-u_{\tilde{N}} \qquad (11)$$

where the parameters c,d controls the filter characteristics. This way, the wavelet filters of an arbitrary order can be derived. The formulas (10), (11) are the extension of the equations (8), (9) and use the same parameters a,b to control the characteristics of the lifting filters. The parameters a,b control the width of the transition bands and the new control parameters c,d control the smoothness the pass and stop bands to prevent the appearance of the lateral lobes: with greater values of a,b the values of c,d tend to be greater. The behavior of the coefficient d is of a rapid increasing, and in practice it has a large values sufficiently high say that the influence of the terms in $H_u(z)$ of the order higher of 3 may be neglected. Thus, in practice, on can use lifting update filter of the order $N=4$ without a significant widening of the update filter transition band: the width of this transition band, mainly, is determined by the lifting predictor frequency characteristics. Fig. 7 shows the frequency responses of (10, 2) lifting filters with $a=28, b=8, c=3$.

III. ADAPTIVE LIFTING ALGORITHM

Experimenting with the images of different nature, we found that the minimum of entropy for the images that consists of plain regions can be obtained with the (2,2) lifting, and, in difference, the images with a large number of small-scale details and abrupt edges are better compressed with the (4,4) filters with the derived coefficients $a=16$ and $b=8$. Naturally, we decide to join both filters in the one adaptive filter, which could be able to perform (2,2) at the plain regions and (4,4) filtering at the vicinity of high local data activity. The resulted adaptive DWT algorithm that uses $\widetilde{N}=2$ and $\widetilde{N}=4$ predictors can be described as follows:

- if the local data activity is small, the decision is made in the favor of $\widetilde{N}=2$ predictor with $a=0$;
- if the local data activity is high, the predictor with $\widetilde{N}=4$ and $a=16$ is applied;
- the update filter characteristics are not change preserving $N=4$ and $b=8$;
- the estimator of local data activity is rather simple: if the magnitude of the difference of the approximations $\left|s_i^{(k-1)}-s_{i-1}^{(k-1)}\right|$ is high, i.e., is greater than a predetermined threshold, the hypothesis is adopted that the local data activity is high and we use the $\widetilde{N}=4$ predictor. Otherwise, the smaller order predictor of $\widetilde{N}=2$ is applied.

Thus, the wavelet lifting decomposition properties are varied between the (4,4) and (2,4) lifting filters. The frequency response of the proposed adaptive filter is varied between the one presented in the Fig. 5 above (4,4 filter) and the presented in the Fig.8 (2,4 filter with $a=0$ and $b=8$).

One can implement adaptive wavelet decomposition with the described algorithm using the lifting filters of higher order, and the lifting filters of a high order can be derived from the generalized lifting scheme (5), (6) with the coefficients given by the presented formulas (10), (11), as it was explained in the previous Section. For example, the (10, 4) wavelet filters represented in Fig. 7 with the parameters $a=28, b=8, c=3$ have more narrow transition bands, ant it is expected these filters would produce better results in energy compactation because of the reduced aliasing comparing to the filters of minor order.

At the restoration stage, the local data activity estimator is applied to the inverse lifted approximations, Because of the properties of the lifting scheme this data are the restored exact copy of the original data that were used by the local data activity estimator. This way, the known problem of synchronization [6] is avoided.

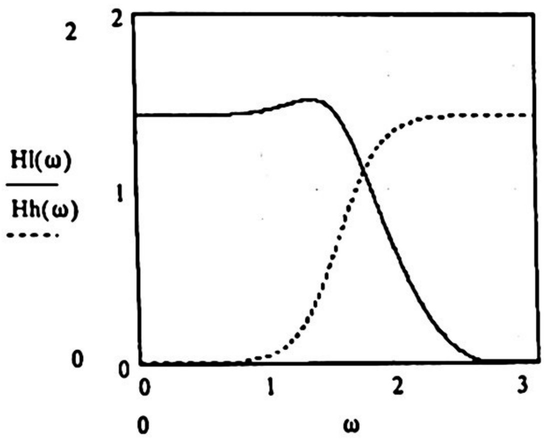

Fig 8. Normalized frequency responses of (2, 4) lifting filters with $a=0$ and $b=8$

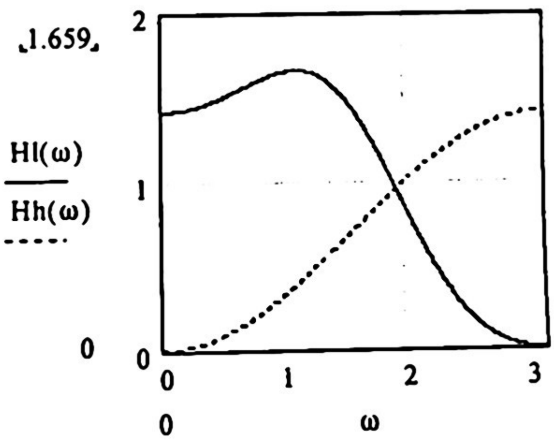

Fig 7. Normalized frequency responses of (10, 4) lifting filters with $a=28, b=8, c=3$

IV. EXPERIMENTAL RESULTS

Table 2 presents the entropy values of the decomposed test images with the different values of the coefficients a, b and for the different values of the threshold in the local data activity estimation and decision making about order of the prediction filter.

It is followed from the result presented in the Table 2 that the presented adaptive spatial-variant algorithm possesses better energy compactation ability comparing to the non-adaptive lifting DWT performance: **4.233**bpp and **6.253**bpp for (2,4) lifting filters with $a=0$, $b=8$, and **4.213**bpp and **6.228**bpp for (4,4) lifting with $a=16$, $b=8$, and **4.214**bpp and **6.226**bpp for (10,4) lifting with $a=16, b=64, c=3$, for the test images "Lena" and "Mandrill", respectively. Surprisingly, the optimal values of the coefficient b are greater than 8, which is the optimal value in the sense of the transition band width and the absence of side lobes in the frequency response of the update lifting filter. This tendency can be explained by the fact that the middle of the transition band of the low-pass update filter is lay too far from the ideal frequency $\frac{\pi}{2}$, and making b greater we reduce the aliasing caused by the non-optimality of the low-pass update filter.

At the same time, if we can make the transition band narrower with a greater a value, the signal energy leaks through the appeared side lobe that results in the increased entropy of the transformed images.

The energy compactation characteristics almost do not depend on the prediction filter order higher than 4, and with the greater coefficient c, and the characteristics of the wavelet lifting predictor in image compression tends to the characteristics of the predictor of order 4.

The value of the threshold for the presented locally adaptive DWT algorithm is varied depending on the image nature: for images with the prevalence of plain regions, this value tends to be small that results in the prevalence of (2,4) filtering; for images with a large number of small scale details and abrupt edges, this value is increased to apply more frequently the predictor of the greater order $\widetilde{N}=4$.

V. CONCLUSION

The novel algorithm of adaptive DWT based on the generalized lifting scheme is presented. The algorithm uses a simple local data activity estimator, which permits to avoid the known synchronization problems. The energy compactation properties of the developed technique are better in comparison to the known wavelet filters that implement the lifting scheme.

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Table 2. Simulation results for the proposed adaptive DWT algorithm

Threshold value	*a*	*b*	(4,4)/(2,4) adaptive filter		(10,4)/(2,4) adaptive filter, c=3	
			"Lena" entropy, bpp	"Mandrill" entropy, bpp	"Lena" entropy, bpp	"Mandrill" entropy, bpp
8	16	8	4.211	6.273	4.212	6.275
		16	4.208	6.265	4.209	6.266
		32	4.203	6.249	4.204	6.251
		48	4.198	6.237	4.198	6.237
		64	4.192	6.226	4.192	6.225
	28	8	4.227	6.311	4.23	6.322
		16	4.224	6.3	4.226	6.311
		32	4.216	6.28	4.216	6.288
16	16	8	4.211	6.266	4.212	6.268
		16	4.21	6.259	4.21	6.26
		32	4.206	6.245	4.206	6.246
		48	4.202	6.234	4.202	6.234
		64	4.197	6.225	4.197	6.224
	28	8	4.213	6.293	4.217	6.302
		16	4.211	6.284	4.211	6.292
		32	4.206	6.267	4.205	6.273
32	16	8	4.222	6.259	4.222	6.26
		16	4.221	6.252	4.221	6.253
		32	4.218	6.241	4.218	6.242
		48	4.215	6.232	4.215	6.232
		64	4.211	6.225	4.211	6.225
	28	8	4.219	6.273	4.219	6.278
		16	4.218	6.265	4.218	6.27
		32	4.216	6.252	4.214	6.257